

LECTURE: 3-7 RATES OF CHANGE IN THE NATURAL AND SOCIAL SCIENCES

Example 1: A particle moves according to the law of motion $s = f(t) = t^4 - 4t + 1$.

(a) Find the velocity at time t . What is the velocity after 2 seconds?

(b) When is the particle at rest?

(c) When is the particle moving forward (in the positive direction)?

(d) Draw a diagram to illustrate the motion of the particle.

(e) Find the total distance traveled in the first 8 seconds.

Example 2: If a ball is thrown vertically upward with a velocity of 80 ft/s, then its height after t seconds is $s = 80t - 16t^2$.

(a) What is the velocity of the ball after 2 seconds?

(b) Sketch a rough graph of the ball's height as a function of time. Using Calculus, find the maximum height reached by the ball.

Example 3: If a tank holds 1000 gallons of water, which drains from the bottom of the tank in 20 minutes, then Torricelli's Law gives the volume of water V remaining in the tank after t minutes as

$$V = 1000 \left(1 - \frac{1}{20}t^2 \right) \quad 0 \leq t \leq 20.$$

Find the rate at which water is draining from the tank after (a) 5 minutes, (b) 10 minutes and (c) 20 minutes. At what time is the water flowing out the fastest? Slowest?

Example 4: The volume of a growing spherical cell is $V = \frac{4}{3}\pi r^3$, where the radius r is measured in micrometers. Find the average rate of change of V with respect to t when r changes from:

(a) 5 to 6 micrometers

(b) 5 to 5.1 micrometers

(c) Find the instantaneous rate of change of V with respect to r when $r = 5$ micrometers.

(d) Show that the rate of change of the volume of a sphere with respect to its radius is equal to its surface area. Why might this be true?

Economics

Marginal Cost Function

Example 5: The cost, in dollars, of producing x yards of a certain fabric is

$$C(x) = 1200 + 12x - 0.1x^2 + 0.0005x^3$$

(a) Find the marginal cost function.

(b) Find $C'(200)$ and explain its meaning. What does it predict?

(c) Compare $C'(200)$ with the cost of manufacturing the 201st yard of fabric.